International Journal of Theoretical Physics, Vol. 37, No. 4, 1998

No-Blueshift Condition in Wolf Mechanism

S. Datta,[1] S. Roy,[1] M. Roy,[2] and M. Moles[3]

Received August, 1997

The idea that quasar redshifts are not cosmological is favored by some astronomers. This means that quasars cannot be assumed to be at very large distances and that the scale of the universe is less than suggested by the big bang cosmology. An alternative mechanism is the Wolf effect, in which light beams interact in a way that can produce frequency shifts. Under suitable choice of source parameters and assuming the existence of strong anisotropy in the interstellar medium, blueshifts can be avoided.

1. THE REDSHIFT CONTROVERSY

We know that the lines observed in the spectra of radiation that reaches us from astronomical sources are shifted relative to those observed in the spectra from the same element on earth. Usually the lines are shifted toward the larger wavelengths and one then speaks of a *redshift.* In a very few cases, they are blueshifted (Wolf, 1991). The relative frequency shift z is defined by the relation

$$z = \frac{\omega_0 - \bar{\omega}_0}{\bar{\omega}_0}$$

where ω_0 and $\bar{\omega}_0$ denote the unshifted and shifted frequency, respectively. From the observed shifts astronomers draw conclusions which have far-reaching consequences for our understanding of the structure of the universe. We briefly state here the conventional or already established explanations of the origin of the redshift.

[1] Physics and Applied Mathematics Unit, Indian Statistical Institute, Calcutta-700035 India.
[2] Department of Physics, University of Milan, Milan, Italy.
[3] Observatorio Astronomico, Nacional Apartado 1143, Madrid, Spain.

0020-7748/98/0400-131 3$15.00/0

1.1. Doppler-like Shift and Expansion of the Universe

If a source is moving with velocity v relative to the observer, then the observer detects a shift $\Delta\lambda$ of a line centered on a wavelength λ, and according to Doppler's formula

$$z = \frac{\Delta\lambda}{\lambda} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} - 1$$

where c is the velocity of light. For positive value of v, *i.e.*, when the source is moving away from the observer, z is positive and can take any value in the range $0 \leq z < \infty$, and then the shift is toward the red. When the source is moving toward the observer, z becomes negative and can take any value between 0 and -1. Figure 1 shows the variation of z with v/c. From the figure it is clear that if $|v|/c << 1$, then the curve is almost linear.

One of the first observations of a redshift was made by Huggins in 1868. Twenty years later Vogel observed $z = 10^{-4}$ from the light coming from the star Capella. It corresponds to a speed of recession of about 30 km/sec. During 1912–1925, at the Lowell observatory V. Slipher analyzed spectra of 40 galaxies and found 38 of them were redshifted and 2 were blueshifted (Wolf, 1991). This fact might protect Wolf's mechanism from Arp's criticism (Arp, 1987). The largest speed of recession among Slipher's observations was about 1800 km/sec. In 1928, while working at Southern California, E. Hubble made a great step in the modern theory of the universe. He noticed that the fainter the galaxy, the larger is its redshift. Hubble assumed that the *faintness of a galaxy is entirely due to its distance* and he gave a simple linear law

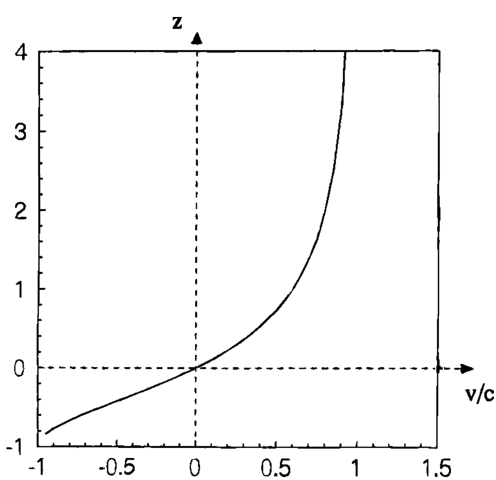

Fig. 1. Variation of z against v/c.

$$v = Hd$$

where v is the speed of recession of the astronomical source and d is its distance from the observer on earth. Here H is a constant, known as Hubble's constant. Its value is a matter of great controversy, since it relates directly to the age of the universe. However, a rough estimate of H is 50 km/sec/megaparsec. Clearly this law provides supporting evidence for the big bang theory.

However, problems have arisen. In 1960, some astronomical objects were found which had very different properties from known stars and galaxies. Astronomers called them quasistellar objects (QSO). Today they are called *quasars*. Their spectral lines are much broader than those found in the spectra of galaxies. The *z numbers* of the lines were very large. For example, $z =$ 0.16 corresponds to a speed of recession of about 15% of the speed of light. Soon afterward quasars with much higher redshifts were found. The implications of these discoveries are being debated.

1. If one assumes the Doppler effect to be the only origin of the observed redshift and that Hubble's law is valid, then the quasars must be at very large distances and therefore must have very high intrinsic luminosities.

2. The age of the universe estimated by big bang cosmology (BBC), *i.e.*, the total elapsed time since the big bang, is given by H^{-1} times a theory-dependent constant of order 1. Uncertainty about the value of H causes several such estimates of the age. According to BBC it is approximated to 17–20 billion years. Quasars with the largest z-number correspond to a speed of recession $v = 0.94c$ (calculated from Doppler's formula). This means that, by virtue of Hubble's law, radiation from the quasar must have originated when the universe was only a few percent of its present age (Wolf, 1991).

3. The luminosities of quasars vary in time with periodicities of the order of a week or a month. Therefore the size of the light-emitting area can be no more than a few lightdays across. Moreover, we can not see even a single ultrabright quasar near the earth, so we are forced to believe that all quasars must have originated long ago, just after the big bang in a 'quasar era' (Drew, 1997)!

4. Finally, some extraordinary photographs have intensified the redshift controversy. The famous photographs of the quasar Markarian 205 and the galaxy NGC 4319 (Kaufmann, 1979; Burbidge, 1988) give clear evidence against BBC. In these photographs we see the quasar interacting with the galaxy. But the most astonishing fact is that the redshift of the quasar is 12 times larger than that of the galaxy. This violates the combined interpretation of Doppler and Hubble.

In connection with the redshift, these are the main features of the Doppler effect and of the expansion of the universe. Another reason for redshift is gravitation.

1.2. Gravitational Redshift

When light passes through a strong gravitational field (for example, a neutron star), this type of redshift is observed. At the time of a total solar eclipse, this shift due to the sun can be calculated. However, the magnitude of this shift is negligibly small. The z-number is given in this case by

$$z = \frac{1}{\sqrt{1 - 2GM/c^2R}} - 1$$

where M and R are the mass and the radius of the star whose gravitational field is under consideration, G is the gravitational constant, and c is the speed of light in vacuum. It is clear from the above equation that z increases if M increases and/or R decreases. Based on this fact, a new concept has been introduced with great importance in preserving BBC against observational and theoretical challenges. It is the *black hole*, which is defined to be an object of mass M having radius smaller than its *Schwarzschild radius* R_S, given by

$$R_S = \frac{2GM}{c^2}$$

Light from an object having radius greater than but close to its Scharzschild radius is highly redshifted. Though there is no direct evidence of black holes, QSOs are assumed to be accompanied by such objects.

These are the three causes of redshift, namely, the *Doppler effect*, the *expansion of the universe*, and *gravitation.* The vast majority of astronomers do not believe that there can be a cause other than these three. Now, over 500 quasars are known, but there is neither a satisfactory model of a QSO, nor a reasonable explanation of its redshift! Today the most important question is: *Apart from these three known causes of redshifts, does there exist another, a* new *one?*

We believe the answer to be positive. Several proposals and explanations are mentioned below.

(i) Recently a mechanism has been proposed by Accardi and co-workers modeling the interstellar medium by a low-density Fermi gas (Accardi *et al.*, 1995). They have proposed a general redshift theorem.

(ii) Arp has shown, over the past two decades, that the quasars appear to cluster near normal galaxies (Arp, 1987).

(iii) Another unexplained (perhaps circumstancial) result is by Tifft, who has suggested that redshifts are *quantized* and they are whole multiples of 72 *km/sec* (Tifft, 1988).

None of the above theories is well developed nor has any scope to be tested experimentally.

An additional a mechanism exists which has no connection with relative motion (Doppler effect/expansion of the universe) or gravitation. In the next section we discuss briefly the main features of this new mechanism for a redshift, called the *Wolf effect.*

2. THE WOLF EFFECT

2.1. Redshift Due to Source Correlation

Let us consider two fluctuating sources located at P_1 and P_2 with ensembles $Q(P_1, \omega)$ and $Q(P_2, \omega)$ representing the fluctuations that generate the field represented by the ensemble $U(P, \omega)$ at a very distant point P (and so it is assumed that $P_1P = P_2P = R_0$). Also we suppose that P_1 and P_2 have identical spectra $S_Q(\omega)$.

Then (Wolf, 1987a)

$$U(P,\omega) = [Q(P_1, \omega) + Q(P_2, \omega)]\,\frac{e^{ikR}}{R_0} \tag{1}$$

The spectrum of light at P is given by

$$S_U(P,\omega) = \langle U^*(P,\omega)U(P,\omega)\rangle \tag{2}$$

where U^* denotes the complex conjugate of U and the angular brackets denote the ensemble average. On simplification, (2) becomes

$$S_U(P,\omega) = \frac{2}{R_0^2}\,S_Q(\omega) + \frac{1}{R_0^2}\,[W_Q(P_1, P_2, \omega) + c.c] \tag{3}$$

where

$$S_Q(\omega) = \langle Q^*(P_1, \omega)Q(P_1, \omega)\rangle = \langle Q^*(P_2, \omega)Q(P_2, \omega)\rangle$$

and

$$W_Q(P_1, P_2, \omega) = \langle Q^*(P_1, \omega)Q(P_2, \omega)\rangle$$

is the cross-spectral density of the source fluctuation; c.c. denotes the complex conjugate. The degree of spectral coherence at frequency ω is given by

$$\mu_Q(P_1, P_2, \omega) = \frac{W_Q(P_1, P_2, \omega)}{S_Q(\omega)} \tag{4}$$

Assuming that $\mu_Q(P_1, P_2, \omega)$ has no preference between P_1 and P_2 and therefore depends on ω only, (3) reduces to

$$S_U(P, \omega) = \frac{2}{R_0^2} S_Q(\omega)[1 + Re\mu_Q(\omega)] \tag{5}$$

Suppose that the spectrum of each of the two sources consists of a single line of *Gaussian profile* (Wolf, 1987b)

$$S_Q(\omega) = A_0 e^{-(\omega-\omega_0)^2/2\delta_0^2} \tag{6}$$

and the correlation between the two sources is characterized by the *degree of spectral coherence* (Wolf, 1987b)

$$\mu_Q(\omega) = ae^{-(\omega-\omega_1)^2/2\delta_1^2} - 1 \tag{7}$$

where A_0, ω_0, δ_0 ($<< \omega_0$), a, ω_1, δ_1 ($<< \omega_1$) are positive constants. Substituting from (6) and (7) in (5), we get after some calculations (Wolf *et al.*, 1989)

$$S_U(P,\omega) = A' e^{-(\omega-\omega_0')^2/2\delta_0'^2} \tag{8}$$

where

$$\begin{aligned} A' &= \frac{2A_0 a}{R_0^2} e^{-(\omega_1-\omega_0)^2/2(\delta_0^2+\delta_1^2)} \\ \omega_0' &= (\delta_0^2\, \omega_1 + \delta_1^2\, \omega_0)/(\delta_0^2 + \delta_1^2) \\ \delta_0'^2 &= (\delta_0^2\, \delta_1^2)/(\delta_0^2 + \delta_1^2) \end{aligned} \tag{9}$$

On the other hand if $\mu_Q = 0$, we have

$$[S_U(P,\omega)]_{\text{uncorr}} = \frac{2A}{R^2} e^{-(\omega-\omega_0)^2/2\delta_0^2} \tag{10}$$

Comparison of equations (10) and (8) shows that although both spectral lines have Gaussian profile, they differ from each other. Clearly from (9) we get $\delta_0' < \delta_0$ and $\omega_0' <$ *or* $> \omega_0$ according as $\omega_1 <$ *or* $> \omega_0$. Hence if $\omega_1 < \omega_0$, the spectrum is redshifted.

Thus the spectrum of light changes on propagation, even through free space. The possibility of producing shifts of spectral lines by this mechanism was first tested by Bocko *et al.* (1987) using acoustic waves. Later several optical rather than acoustical experiments were performed (Morris and Faklis, 1987).

2.2. Redshift Due to Scattering through Random Medium

Wolf has suggested another mechanism which can produce a considerable amount of redshift. However, the success of this theory entirely depends on the characteristics of the interstellar medium, about which very little is known. Here we describe briefly the main results given by James and Wolf (1990) with a little generalization by taking polychromatic instead of monochromatic light. Suppose that a polychromatic field of central frequency ω_0 is incident in the direction specified by the unit vector $u = (u_x, u_y, u_z)$ on a scattering medium. The incident spectrum then has the form

$$S_U(\omega) = A_0 \exp\left[-\frac{1}{2\delta_0^2} (\omega - \omega_0)^2 \right] \tag{11}$$

The spectrum of the scattered radiation at a point $r\hat{u}'$ in the far zone produced when a linearly polarized polychromatic plane electromagnetic wave is incident on such a medium was shown to be given by the following formula (Wolf and Foley, 1989) valid within the first-order Born approximation (Wolf and Born, 1980; Goodman, 1985)

$$S^{(\infty)}(\omega') = A\omega'^4 \int_{-\infty}^{\infty} \mathcal{K}(\omega', \omega) S_U(\omega)\, d\omega \tag{12}$$

Here $A = (2\pi)^3 V(\sin^2\psi)/c^4 r^2$, ψ being the angle between the electric vector of the incident field and $\hat{u}' = (u'_x, u'_y, u'_z)$, the unit vector in the direction of scattering; V is the volume of the scatterer, c is the speed of light, and $\mathcal{K}$ is the scattering kernel defined as (Wolf and Foley, 1989)

$$\mathcal{K}(\omega', \omega) = \bar{G}\left(\frac{\omega'\hat{u}' - \omega\hat{u}}{c}, \omega' - \omega; \omega \right) \tag{13}$$

where $\bar{G}(\bar{\vec{K}}, \Omega, \omega)$ is the four-dimensional Fourier transform of the correlation function

$$G(\bar{\vec{R}}, T; \omega) = \langle \eta^*(\bar{\vec{r}} + \bar{\vec{R}}, t + T; \omega)\eta(\bar{\vec{r}}, t; \omega) \rangle \tag{14}$$

of the generalized dielectric susceptibility $\eta(\vec{r}, t; \omega)$ of the scattering medium. Now we study a particular case supposing that the correlation properties of the fluctuating medium are characterized by an anisotropic Gaussian function, viz.,

$$G(\vec{R}, T; \omega) = G_0 \exp\left[-\frac{1}{2}\left(\frac{X^2}{\sigma_x^2} + \frac{Y^2}{\sigma_y^2} + \frac{Z^2}{\sigma_z^2} + \frac{c^2T^2}{\sigma_\tau^2} \right) \right] \tag{15}$$

Here (X,Y,Z) are components of the vector $\vec{R}$ with respect to a suitably chosen

Cartesian reference frame; σ_x, σ_y, σ_z, σ_τ are correlation lengths and G_0 is a positive constant. The Fourier transform of $G(\vec{R}, T; \omega)$ is given by

$$G(\vec{K}, \Omega; \omega) = \frac{1}{(2\pi)^4} \int_V d^3R \int_{-\infty}^{\infty} dT\, G(\vec{R}, T; \omega) \exp[-i(\vec{K}\cdot\vec{R} - \Omega T)] \quad (16)$$

$$= B \exp\left[-\frac{1}{2}\left(\sigma_x^2 K_x^2 + \sigma_y^2 K_y^2 + \sigma_z^2 K_z^2 + \frac{\sigma_\tau^2 \Omega^2}{c^2}\right)\right]$$

where

$$B = \frac{G_0 \sigma_x \sigma_y \sigma_z \sigma_\tau}{c(2\pi)^2}$$

and $\vec{K} = (K_x, K_y, K_z)$ with the same reference frame as that of $\vec{R}$. Using (13), we get

$$\mathcal{K}(\omega', \omega) = B \exp\left[-\tfrac{1}{2}(\alpha'\omega'^2 - 2\beta\omega\omega' + \alpha\omega^2)\right] \quad (17)$$

where

$$\alpha = \frac{\sigma_x^2}{c^2} u_x^2 + \frac{\sigma_y^2}{c^2} u_y^2 + \frac{\sigma_z^2}{c^2} u_z^2 + \frac{\sigma_\tau^2}{c^2}$$

$$\alpha' = \frac{\sigma_x^2}{c^2} u_x'^2 + \frac{\sigma_y^2}{c^2} u_y'^2 + \frac{\sigma_z^2}{c^2} u_z'^2 + \frac{\sigma_\tau^2}{c^2} \quad (18)$$

$$\beta = \frac{\sigma_x^2}{c^2} u_x u_x' + \frac{\sigma_y^2}{c^2} u_y u_y' + \frac{\sigma_z^2}{c^2} u_z u_z' + \frac{\sigma_\tau^2}{c^2}$$

According to Schwarz's inequality,

$$\alpha\alpha' \geq \beta^2$$

the equality applies only when $\hat{u} \| \hat{u}'$.

Performing a straightforward calculation, we obtain from equations (12) and (17),

$$S^{(\infty)}(\omega') = A' \exp\left[-\frac{1}{2\delta_0'^2}(\omega' - \bar{\omega}_0)^2\right] \quad (19)$$

where

$$\left.\begin{aligned} \bar{\omega}_0 &= \frac{|\beta|\omega_0}{\alpha' + \delta_0'^2(\alpha\alpha' - \beta^2)} \\ \delta_0'^2 &= \frac{\alpha\delta_0^2 + 1}{\alpha' + \delta_0^2(\alpha\alpha' - \beta^2)} \\ A' &= \sqrt{\frac{\tau}{2(\alpha\delta_0'^2 + 1)}}\, ABA_0\omega_0'^4\delta_0 \exp\left[\frac{|\beta|\omega_0\bar{\omega}_0 - \alpha\omega_0^2}{2(\alpha\delta_0^2 + 1)}\right] \end{aligned}\right\} \qquad (20)$$

To a good approximation (James and Wolf, 1990) we can replace ω' in A' by $\bar{\omega}_0$ defined in the first equation of (20) so that we can consider A' as a constant. Thus equation (19) suggests that $S^{(\infty)}(\omega')$ has also the form of a spectral line of Gaussian profile, with central frequency $\bar{\omega}_0$.

In this case the relative frequency shift is given by

$$z = \frac{\alpha' + \delta_0'^2(\alpha\alpha' - \beta^2)}{|\beta|} - 1 \qquad (21)$$

Thus we see that the relative frequency shift z induced by this mechanism is independent of frequency and can take values in the range $z > -1$ even though the source, the medium, and the observer are at rest with respect to one another. It is therefore necessary to consider these effects which can contribute a large amount to the redshift of the observed spectra. Thus this redshift mechanism, of non-Doppler origin might play a significant role in testing cosmological models.

3. NO-BLUESHIFT CONDITION

In the case of source correlation this condition is very simple. We see from (9) that the spectrum of the field differs from the spectrum of the source, depending on the degree of correlation. If the mean frequency of the degree of spectral coherence is less than that of the source, *i.e.*, $\omega_1 < \omega_0$, then there will be no blueshift and the spectrum will be redshifted.

On the other hand, the 'no-blueshfft condition' for the case of scattering through a random medium is rather complicated. From (21) we get that for a redshift, $\alpha' + \delta_0'^2(\alpha\alpha' - \beta^2)$ must be greater than $|\beta|$. This means that the 'no-blueshift condition' entirely depends on the medium characteristics. Since we have no standard model of the interstellar medium, no definite values of the medium parameters implicitly present in the above condition can be fixed. However, we consider here two special cases:

1. If we take an isotropic medium, *i.e.*,

$$\sigma_x = \sigma_y = \sigma_z$$

then,

$$\alpha = \frac{\sigma_x^2}{c^2} + \frac{\sigma_\tau^2}{c^2}$$

$$\alpha' = \frac{\sigma_x^2}{c^2} + \frac{\sigma_\tau^2}{c^2}$$

$$\beta = \frac{\sigma_x^2}{c^2} \cos\theta + \frac{\sigma_\tau^2}{c^2}$$

where θ is the angle of scattering. Then

$$\alpha' + \delta_0'^2(\alpha\alpha' - \beta^2) - |\beta| = \frac{\sigma_x^2}{c^2}(1 - |\cos\theta|) + \delta_0'^2(\alpha\alpha' - \beta^2) \geq 0$$

where in the last step we apply $|cos\ \theta| \leq 1$ and Schwarz's inequality. Thus in an isotropic medium there can only be a redshift.

2. We now assume that

$$\sigma_x, \sigma_\tau >> \sigma_{\bar{y}}, \sigma_z,$$

so that we can neglect σ_y^2 and σ_z^2 compared to σ_x^2 and σ_τ^2. Then

$$\alpha = \frac{\sigma_x^2}{c^2} u_x^2 + \frac{\sigma_\tau^2}{c^2}$$

$$\alpha' = \frac{\sigma_x^2}{c^2} u_x'^2 + \frac{\sigma_\tau^2}{c^2} \tag{22}$$

$$\beta = \frac{\sigma_x^2}{c^2} u_x u_x' + \frac{\sigma_\tau^2}{c^2}$$

Then

$$\alpha' + \delta_0'^2(\alpha\alpha' - \beta^2) - |\beta|$$

$$= \frac{\sigma_x^2}{c^2}(u_x^2 - |u_x u_x'|) + \delta_0'^2(\alpha\alpha' - \beta^2)$$

$$= \frac{\sigma_x^2}{c^2}|u_x|(|u_x| - |u_x'|) + \sigma_0'^2(\sigma\sigma' - \beta^2)$$

$$\geq 0 \qquad \text{if} \quad |u_x| \geq |u_x'| \qquad \text{(applying Schwarz's inequality)}$$

Note that it is only a sufficient condition, not a necessary one. The physical meaning of this condition may be stated as follows. We imagine the observer's sky as a unit hemisphere, and we plot (more precisely, project) the observed quasars on it. We then model the interstellar medium as a random medium with correlation property characterized by an anisotropic Gaussian function

$$G(\bar{R}, T; \omega) = G_0 \exp\left[-\frac{1}{2}\left(\frac{X^2}{\sigma_x^2} + \frac{Y^2}{\sigma_y^2} + \frac{Z^2}{\sigma_z^2} + \frac{c^2T^2}{\sigma_\tau^2}\right)\right]$$

the correlation lengths σ_x, σ_y, σ_z are chosen as $\sigma_x, \sigma_\tau >> \sigma_y, \sigma_z$, so that we can neglect σ_y^2 and σ_z^2 compared to σ_x^2 and σ_τ^2. Moreover, the direction around which the local density of the points (quasars) plotted on the surface of the hemisphere is maximum is taken to be the direction of σ_x; *i.e.*, the direction of the greatest correlation length.

We now see how the z–θ relation changes with the anisotropy. For this we introduce here a new parameter k which is defined by the ratio of θ_x with σ_τ,

$$k = \frac{\sigma_x}{\sigma_\tau}$$

Now substituting (22) in (21), we get

$$z = \left\{\left(\frac{\sigma_x^2}{c^2} u_x'^2 + \frac{\sigma_\tau^2}{c^2}\right) + \delta_0'^2\left[\left(\frac{\sigma_x^2}{c^2} u_x^2 + \frac{\sigma_\tau^2}{c^2}\right)\left(\frac{\sigma_x^2}{c^2} u_x^2 + \frac{\sigma_\tau^2}{c^2}\right) - \left(\frac{\sigma_x^2}{c^2} u_x u_x' + \frac{\sigma_\tau^2}{c^2}\right)^2\right]\right\}\left\{\left|\frac{\sigma_x^2}{c^2} u_x u_x' + \frac{\sigma_\tau^2}{c^2}\right|\right\}^{-1} - 1$$

Dividing both numerator and denominator by σ_τ^2/c^2, (we get

$$z = \frac{(k^2 + 1) + Mk^2(u_x - u_x')^2}{|k^2 u_x u_x' + 1|} - 1$$

where $M = (\sigma_\tau^2/c^2)\delta_0'^2$. For $u_x' = 1$ (*i.e.*, the direction of the telescope is taken as the x axis)

$$z = \frac{(k^2 + 1) + Mk^2(u_x - 1)^2}{|k^2 u_x + 1|} - 1$$

For $M \to 0$, z is approximately given by

$$z = \frac{k^2 + 1}{|k^2 u_x + 1|}$$

Moreover, $cos(\theta) = \hat{u}\hat{u}^1 = u_x$. And since θ is very small (positive or negative), $cos(\theta) > 0$. Hence,

$$z = \frac{k^2 + 1}{k^2\, cos(\theta) + 1}$$

In the Fig. 2 we plot the z–θ relation for two values of k.

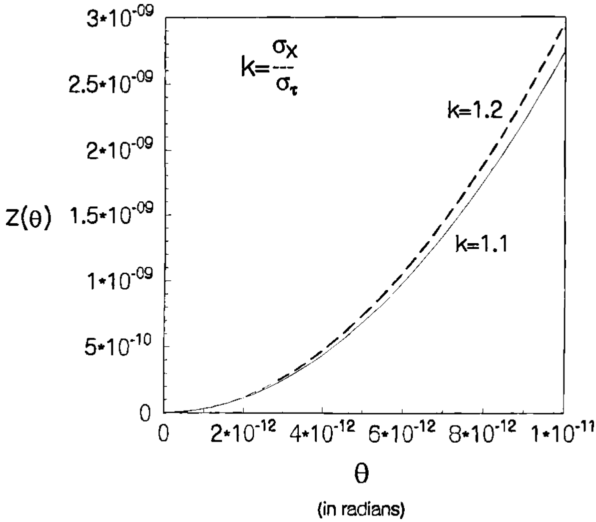

Fig. 2. Variation of z against θ for two values of k.

4. DISCUSSION

In our previous work, we have shown the joint effect of Wolf's mechanisms for producing frequency shifts, *viz.*, due to source correlation and due to random scattering medium, and also the effect of multiple scattering. In those mechanisms, though the possibilities of a blueshift are discussed, they were not prominent enough that we should have to discard the whole theory. In fact, we show here that the mechanism agrees very much with the observational results. Even if a little amount of blueshift is produced in such processes, the redshifts due to other causes dominate its effect on the observed shift. As a result, the observed quasarshifts are always toward the red. We explain this below.

In our previous paper (Datta *et al.*, n.d.) we formulated the general rule for combining shifts from several origins. The final shift is given by

$$z = (1 + z_1)(1 + z_2) - 1$$

where z_1 and z_2 are two primary shifts, one of them being red and the other blue (say, z_2). Moreover, let $|z_1| > |z_2|$. Then

$$\begin{aligned} z &= |z_1| - |z_2| - |z_1 z_2| \\ &= |z_1| - |z_2|(1 - |z_1|) \\ &> 0 \qquad \text{since} \quad 1 - |z_1| \text{ can never exceed } 1 \end{aligned}$$

Indeed, the blueshifts due to Wolf's mechanism are very small compared to the redshifts. Therefore the observer finds the quasars always to be redshifted. However, more information regarding the structure of the interstellar medium is necessary to justify our no-blueshift condition in the Wolf framework.

ACKNOWLEDGMENT

We are grateful to Prof. Emil Wolf, University of Rochester, for his valuable opinions regarding our work and also for his constant inspiration.

REFERENCES

Accardi, L., Laio, A., Lu, Y. G., and Rizzi, G. (1995). *Physics Letters A*, **209**, 277.
Arp, H. (1987). *Quasars, Redshifts, and Controversies*, Interstellar Media, Berkeley, California.
Bocko, M. F., Douglass, D. H., and Knox, R. S. (1987). *Physical Review Letters*, **58**, 2649.
Burbidge, G. (1988). *Mercury*, **17**, 136.
Datta, S., Roy, S., and Roy, M. (n.d.). Frequency shift of spectral lines generated by multiple dynamic scattering, submitted.
Drew, H. R. (1997). On the nature of things . . ., *Apeiron*, **4**, 26.
Goodman, J. W. (1985). *Statistical Optics*, Wiley, New York.
James, D. F. V., and Wolf, E. (1990). *Physics Letters A*, **146**, 167.
Kaufmann, William J. (1979). *Galaxies and Quasars*, Freeman, San Francisco.
Morris, G. M., and Faklis, D. (1987). *Optics Communications*, **62**, 5.
Tifft, W. G. (1988). In *New Ideas in Astronomy*, F. Bertola, J. W. Sulentic, and B. F. Madore, eds., Cambridge University Press, Cambridge.
Wolf, E. (1987a). *Nature*, **326**, 363.
Wolf, E. (1987b). *Physical Review Letters*, **58**, 2646.
Wolf, E. (1991). The redshift controversy and . . ., *NPL Technical Bulletin*, October 1991.
Wolf, E., and Born, M. (1980). *Principles of Optics*, 6th ed., Pergamon Press, Oxford.
Wolf, E., and Foley, J. T. (1989). *Physical Review A*, **40**, 579.
Wolf, E., Foley, J. T., and Gori, F. (1989). *Journal of the Optical Society of America A*, **6**, 1142.